Motion Due to Gravity

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The first person to state clearly that all objects on Earth fall with the same acceleration was Galileo (1564 – 1642). He used experimental observation with mathematical argument to arrive at this.

This acceleration, denoted by the letter g, is known as the **acceleration due to gravity.**
It has a value of approximately 9.81 m s^{-2}, which will be used in this leaflet.
(In practice, 9.8 m s^{-2} or even 10 m s^{-2} are also used)

Worked Example 1.

If an object, of mass m, is falling under the action of gravity, as in Figure 1, what is the magnitude of the force W on the object?

Solution

Considering Figure 1 and using Newton's Second Law of Motion:

$$\begin{aligned} F &= ma \\ \Rightarrow \mathrm{W} &= mg \end{aligned}$$

This resultant force is the object's weight and has magnitude mg.

Figure 1

The weight, W, of an object, of mass m, is its mass $\times$ gravity, mg.

Worked Example 2.

A hotel lift is taking some guests from the ground floor to the second floor. The guests and the lift combined have a mass of 1400 kg. If the lift accelerates upwards at 1.4 m s^{-2}, what is the tension in the lift cable? (Take $g = 9.81$ m s^{-2})

Solution

Modelling the guests and lift as a single particle, the diagram with forces on is as shown in Figure 2. The resultant force is T $-$ 1400g.

Using Newton's Second Law of Motion:

$$\begin{aligned} F &= ma \\ \mathrm{T} - 1400g &= 1400 \times 1.4 \\ \Rightarrow \mathrm{T} &= 1960 + 13734 = 16000 \text{ N (2s.f.)} \end{aligned}$$

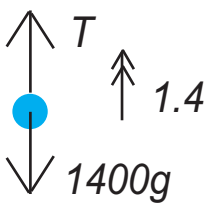

Figure 2

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In the first two examples there has been no mention of a resistive force. In practice, especially when modelling an object falling under gravity, there will almost certainly be such a force and this is usually **air resistance**. However, in many examples people state as part of their assumptions that air resistance will be neglected.

For a given object, a formula for air resistance, R, is usually found experimentally. For low speeds R $= k_1 v$ is commonly used and for higher speeds R $= k_2 v^2$ is used (where v is the speed of the object and k_1 and k_2 are constants that cover all the other factors affecting air resistance).

Worked Example 3.

A conker, of mass 0.2 kg, falls vertically down from a tree in Autumn. Whilst it falls it experiences air resistance of magnitude $0.4v$, where v is its speed in m s^{-1}. Calculate the speed at which it is falling when it has an acceleration of 1.81 m s^{-2} (Take $g = 9.81$ m s^{-2}).

Solution

Modelling the conker as a particle, as in Figure 3, the resultant force is $0.2g - 0.4v$. Then, using Newton's Second Law of Motion:

$$\begin{aligned} F &= ma \\ 0.2g - 0.4v &= 0.2 \times 1.81 \\ 1.962 - 0.4v &= 0.362 \\ 1.6 &= 0.4v \\ \Rightarrow v &= 4.0 \text{ m s}^{-1} \end{aligned}$$

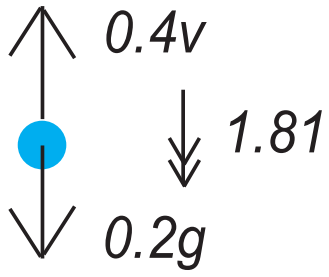

Figure 3

Exercises

1. If an object, of mass 5 kg, is falling under the action of gravity, g, what is the net force on the object?
2. A hotel lift is taking some guests from the first floor to the fifth floor. The guests and the lift combined have a mass of 1250 kg. If the lift accelerates upwards at 1.6 m s^{-2}, what is the tension in the lift cable?
3. A conker, of mass 0.15 kg, falls vertically down from a tree in Autumn. Whilst it falls it experiences air resistance of magnitude $0.2v$, where v is its speed in m s^{-1}. Calculate the speed it is falling at when it has an acceleration of 5.81 m s^{-2}
4. A cricket ball has a mass of 1.2 kg. What is its weight?
5. A shopping centre lift is taking some people from the first floor to the ground floor. Given the tension in the lift cable is 12400 N and the acceleration downwards is 1.81 m s^{-2}, what is the total mass of the people and the lift?
6. A sponge, of mass 0.25 kg, accidently falls vertically down from a window cleaner's hand when he is cleaning a high rise office block. It experiences air resistance, R, given by R $= 0.15v^2$, where v is the speed in m s $^{-1}$. Calculate the sponge's acceleration when it is falling at 3 m s^{-1}.

Answers (all to 2 s.f.)

1. 49 N 2. 14000 N 3. 3.0 m s $^{-2}$ 4. 12 N 5. 1600 kg 6. 4.4 m s $^{-2}$

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