Time Series

A time series Y_t $(t = 1, 2, \ldots, n)$ is a set of n observations recorded through time t, (e.g. days, weeks, months). The arithmetic mean of blocks of k successive values

$$\frac{Y_1 + Y_2 + \ldots + Y_k}{k}, \frac{Y_2 + Y_3 + \ldots + Y_{k+1}}{k}, \ldots$$

is a **simple moving average** of order k, itself a time series which is *smoother* than Y_t and can be used to track, or estimate, the underlying level, μ_t, of Y_t. If $0 < \alpha < 1$ an **exponentially weighted moving average** (EWMA) at time t uses a discounted weighted average of current and past data to estimate μ_t with

$$\hat{\mu}_t = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + \ldots$$

This is equivalent to the recurrence relation

$$\hat{\mu}_t = \alpha Y_t + (1-\alpha)\hat{\mu}_{t-1}$$

Moving averages are often plotted on the same graph as Y_t. If Y_t additionally contains trend, R_t, the rate of change of data per unit time, and $\mu_t = \mu_{t-1} + R_{t-1}$, then the recurrence relation is

$$\hat{\mu}_t = \alpha Y_t + (1-\alpha)(\hat{\mu}_{t-1} + \hat{R}_{t-1})$$

If $0 < \beta < 1$ the *trend smoothing equation* is

$$\hat{R}_t = \beta(\hat{\mu}_t - \hat{\mu}_{t-1}) + (1-\beta)\hat{R}_{t-1}$$

known as *Holt's Linear Exponential Smoothing*.

If Y_t also contain *seasonality*, S_t, a smoothing constant γ, $(0 < \gamma < 1)$ is used in a *seasonal smoothing equation*,

$$\hat{S}_t = \gamma Y_t/\hat{\mu}_t + (1-\gamma)\hat{S}_{t-k},$$

assuming the periodicity is k, with *multiplicative* seasonality. For monthly data $k = 12$.

Forecasting from time n (now) to time $n+h$ ($h = 1, 2, \ldots$)

Level only, $\hat{Y}_{n+h} = \hat{\mu}_n$, the latest EWMA.

Level and constant trend, $\hat{Y}_{n+h} = a + b(n+h)$, the simple linear regression trend line of Y_t against t.

Level and changing trend, $\hat{Y}_{n+h} = \hat{\mu}_n + h\hat{R}_n$.

Level, changing trend and seasonality, $\hat{Y}_{n+h} = \hat{\mu}_n + h\hat{R}_n$, where $\hat{\mu}_n = \alpha Y_n/\hat{S}_{n-12} + (1-\alpha)(\hat{\mu}_{n-1} + \hat{R}_{n-1})$.