## Standard statistical distributions

| Name/parameters | Conditions/application | pdf/pmf | Mean | Variance | mgf | Notes |
|---|---|---|---|---|---|---|
| Binomial<br>Bin$(n,p)$<br>Positive integer $n$<br>Probability $p$, $0 \le p \le 1$ | $n$ independent success/fail trials each with probability $p$ of success. $X$ = number of successes. | $P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$<br>$x=0,1,\ldots,n$ | $np$ | $np(1-p)$ | $\left(1-p+pe^t\right)^n$ | $X \sim$ Bin $(n,p)$<br>$\Rightarrow n-X \sim$ Bin$(n,1-p)$ |
| Geometric<br>Geom$(p)$<br>Probability $p$, $0 \le p \le 1$ | Repeated independent success/fail trials each with probability $p$ of success. $X$ = number of trials up to and including the first success. | $P(X=x)=(1-p)^{x-1}p$<br>$x=1,2,\ldots$ | $\dfrac{1}{p}$ | $\dfrac{1-p}{p^2}$ | $\dfrac{pe^t}{1-(1-p)e^t}$ | Has the "lack of memory" property<br>$P(X>a+b\,\vert\,X>b)=P(X>a)$ |
| Poisson<br>Po$(\lambda)$<br>$\lambda$ a positive number | Events occur randomly at a constant rate. $X$ = number of occurrences in some interval. $\lambda$ is the expected number of occurrences | $P(X=x)=e^{-\lambda}\dfrac{\lambda^x}{x!}$<br>$x=0,1,2,\ldots$ | $\lambda$ | $\lambda$ | $\exp(\lambda(e^t-1))$ | Useful as approximation to Bin$(n,p)$ if $n$ is large and $p$ is small |
| Normal<br>$N(\mu,\sigma^2)$<br>$\mu$, $\sigma$ both real; $\sigma>0$ | A widely used distribution for symmetrically distributed random variables with mean $\mu$ and standard deviation $\sigma$. | $f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left(-\dfrac{(x-\mu)^2}{2\sigma^2}\right)$<br>all real $x$ | $\mu$ | $\sigma^2$ | $\exp\left(\mu t+\dfrac{1}{2}\sigma^2t^2\right)$ | Can approximate Binomial, Poisson, Pascal and Gamma distributions (see Central Limit Theorem) |
| Exponential<br>Expon$(\theta)$ | Events are occurring at rate $\theta$ per unit time. $X$ = time to first occurrence. | $f(x)=\theta\exp(-\theta x)$<br>$x>0$ | $\dfrac{1}{\theta}$ | $\dfrac{1}{\theta^2}$ | $\dfrac{\theta}{\theta-t}$, $t<\theta$ | Has the "lack of memory" property<br>$P(X>a+b\,\vert\,X>b)=P(X>a)$ |
| Negative-binomial or Pascal<br>Pasc$(r,p)$<br>Positive integer $n$<br>Probability $p$, $0 \le p \le 1$ | Repeated independent success/fail trials each with probability $p$ of success. $X$ = number of trials up to and including the $r$-th success. | $P(X=x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}$<br>$x=r,r+1,r+2,\ldots$ | $\dfrac{r}{p}$ | $\dfrac{r(1-p)}{p^2}$ | $\left(\dfrac{pe^t}{1-(1-p)e^t}\right)^r$ | Pasc$(1,p)\equiv$ Geom$(p)$ |
| Gamma<br>Ga$(\alpha,\beta)$<br>$\alpha,\beta>0$ | Generalization of the exponential distribution; if $\alpha$ is an integer it represents the waiting time to the $\alpha$-th occurrence of a random event where $\beta$ is the expected number of events. | $f(x)=\dfrac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$<br>$x>0$ | $\dfrac{\alpha}{\beta}$<br>$\alpha>1$ | $\dfrac{\alpha}{\beta^2}$ | $\left(\dfrac{\beta}{\beta-t}\right)^\alpha$, $t<\beta$ | Ga$(1,\lambda)\equiv$ Expon$(\lambda)$<br>If $\nu$ is an integer, Ga$(\nu/2,2)$ is $\chi^2_\nu$, the Chi-squared distribution with $\nu$ df. |