Permutations and combinations

The number of ways of selecting r objects out of a total of n, where the order of selection is important, is the number of **permutations**: ${}^nP_r = \dfrac{n!}{(n-r)!}$. The number of ways in which r objects can be selected from n when the order of selection is not important is the number of **combinations**:

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

${}^nC_n$ must equal 1, so $0! = 1$ and ${}^nC_0 = 1$; ${}^nC_r = {}^nC_{n-r}$. Also

$${}^nC_0 + {}^nC_1 + \ldots {}^nC_{n-1} + {}^nC_n = 2^n$$
$${}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$$

Random variables

Data arise from observations on variables that are **measured** on different **scales**. A *nominal* scale is used for named categories (e.g. race, gender) and an *ordinal* scale for data that can be ranked (e.g. attitudes, position) - no arithmetic operations are valid with either. *Interval* scale measurements can be added and subtracted only (e.g. temperature), but with *ratio* scale measurements (e.g. age, weight) multiplication and division can be used meaningfully as well. Generally, random variables are either *discrete* or *continuous*. Note: in reality, all data are discrete because the accuracy of measuring is always limited.

A **discrete** random variable X can take one of the values $x_1, x_2, \ldots,$, the probabilities $p_i = P(X = x_i)$ must satisfy $p_i \geq 0$ and $p_1 + p_2 + \ldots = 1$. The pairs (x_i, p_i) form the **probability mass function** (pmf) of X.

A **continuous** random variable X takes values x from a continuous set of possible values. It has a **probability density function** (**pdf**) $f(x)$ that satisfies $f(x) \geq 0$ and

$\int f(x)\mathrm{d}x = 1$, with $P(a < x \leq b) = \displaystyle\int_a^b f(x)\mathrm{d}x$.

Expected values

The expected value of a function $H(X)$ of a random variable X is defined as

$$E\left[H(X)\right] = \begin{cases} \sum H(x_i)P(X = x_i), & X \text{ discrete.} \\ \int H(x)f(x)\mathrm{d}x, & X \text{ continuous.} \end{cases}$$

Expectation is linear in that the expectation of a linear combination of functions is the same linear combination of expectations. For example,

$$E[X^2 + \log X + 1] = E[X^2] + E[\log X] + 1$$

but

$$E[\log X] \neq \log E[X] \text{ and } E[1/X] \neq 1/E[X]$$