Variance

The variance of a random variable is defined as

$$\text{Var}(X) = \mathrm{E}[(X-\mu)^2] \equiv \mathrm{E}[X^2] - \mu^2$$

Properties:

Var$(X) \geq 0$ and is equal to 0 only if X is a constant.
Var$(aX+b) = a^2$Var(X), where a and b are constants.

Moment generating functions

The moment generating function (mgf) of a random variable is defined as

$$M_X(t) = \mathrm{E}[\exp(tX)] \qquad \text{if this exists.}$$

$\mathrm{E}[X^k]$ can be evaluated as the:
(i) coefficient of $\dfrac{t^r}{r!}$ in the power expansion of $M_X(t)$.
(ii) r-th derivative of $M_X(t)$ evaluated at $t=0$.

Measures of location

The **mean** or **expectation** of the random variable X is $\mathrm{E}[X]$, the long-run average of realisations of X. The **mode** is where the **pmf** or **pdf** achieves a maximum (if it does so). For a random variable, X, the **median** is such that $P(X \leq \text{median}) = \frac{1}{2}$, so that 50% of values of X occur above and 50% below the median.

Percentiles

x_p is the 100-p-th percentile of a random variable X if $P(X \leq x_p) = p$. For example, the 5th percentile, $x_{0.05}$, has 5% of the values smaller than or equal to it. The **median** is the 50-th percentile, the **lower quartile** is the 25th percentile, the **upper quartile** is the 75th percentile.

Measures of dispersion

The **inter-quartile range** is defined to be the difference between the upper and lower quartiles, UQ - LQ. The **standard deviation** is defined as the square root of the variance, $\sigma = \sqrt{\mathrm{Var}(X)}$, and is in the same units as the random variable X.

Cumulative Distribution Function

This is defined as a function of any real value t by

$$F(t) = P(X \leq t)$$

If X is a continuous random variable, F is a continuous function of t; if X is discrete, then F is a step function.