Grouped Frequency Data

If the data are given in the form of a grouped frequency distribution where we have f_i observations in an interval whose mid-point is x_i then, if $\sum f_i = n$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{n} \qquad \text{and}$$

$$S_{xx} = \sum f_i (x_i - \bar{x})^2 = \sum f_i x_i^2 - \frac{\left(\sum f_i x_i\right)^2}{n}.$$

Events & probabilities

The *intersection* of two events A and B is $A \cap B$. The union of A and B is $A \cup B$. A and B are **mutually exclusive** if they cannot both occur, denoted $A \cap B = \emptyset$ where $\emptyset$ is called the **null event**. For an event A, $0 \leq P(A) \leq 1$. For two events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B are mutually exclusive then

$$P(A \cup B) = P(A) + P(B).$$

Equally likely outcomes

If a complete set of n elementary outcomes are all equally likely to occur, then the probability of each elementary outcome is $\frac{1}{n}$. If an event A consists of m of these n elements, then $P(A) = \frac{m}{n}$.

Independent events

A, B are *independent* if and only if $P(A \cap B) = P(A)P(B)$.

Conditional Probability of A given B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \text{if} \quad P(B) \neq 0.$$

Bayes' Theorem: $\quad P(B|A) = \dfrac{P(A|B)P(B)}{P(A)}.$

Theorem of Total Probability

The k events B_1, B_2, $\ldots B_k$ form a *partition* of the sample space S if $B_1 \cup B_2 \cup B_3 \ldots \cup B_k = S$ and no two of the B_i's can occur together. Then $P(A) = \sum_i P(A|B_i)P(B_i)$. In this case Bayes' Theorem generalizes to

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_j P(A|B_j)P(B_j)} \qquad (i = 1, 2, \ldots k)$$

If B' is the *complement* of the event B , $P(B') = 1 - P(B)$ and $P(A) = P(A|B)P(B) + P(A|B')P(B')$ is a special case of the theorem of total probability. The complement of the event B is commonly denoted $\overline{B}$.