12. Impulse & Momentum

Linear momentum, $\underline{p}$, of a body of mass, m, with velocity, $\underline{v}$, is a vector quantity defined as $\underline{p} = m\underline{v}$.

Impulse: If a constant force, $\underline{F}$, acts over a time, t, on the body then the impulse of the force is defined as Impulse $= \underline{F}\,t$. Impulse is a vector quantity. The unit of impulse is the same as the unit of momentum.

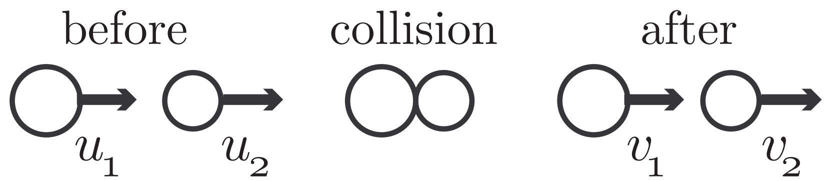
Relationship between momentum and impulse: If a force acts on a body over a time t, the impulse of the force equals the final momentum minus the initial momentum. For the case of a constant force,

$$\underline{F}t = m\underline{v} - m\underline{u}$$

Principle of conservation of linear momentum: When no resultant external force acts on a system of interacting (colliding) particles the total momentum of the system remains constant.

The collision of two bodies: An **elastic** collision is one in which the total kinetic energy is conserved. An **inelastic** collision is one in which the total kinetic energy always decreases.

Consider the collision between two spheres moving in the same line.



Let

m_1, m_2 = the masses of the two spheres
u_1, u_2 = the velocities before collision
v_1, v_2 = the velocities after collision
$v_a = u_1 - u_2$ = the speed of approach
$v_s = v_2 - v_1$ = the speed of separation

In a collision v_a and v_s are connected by the relation

$$v_s = e\,v_a, \quad \text{or} \quad v_2 - v_1 = e(u_1 - u_2)$$

where $0 \le e \le 1$ and is called the **coefficient of restitution**.

In an elastic collision, $e = 1$. For an elastic collision

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

In the case of spheres having the same mass ($m_1 = m_2$)

$$u_2 = v_1, \quad u_1 = v_2$$

which means the spheres exchange velocities.

In a 'perfectly inelastic' collision, where the bodies coalesce, $e = 0$. Then $v_1 = v_2$; there is no rebound, as shown.