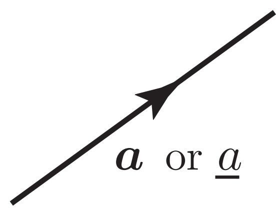
1. Vectors

Force, velocity and acceleration which involve both a magnitude and direction, are **vectors**. A vector is written using a bold typeface, $\boldsymbol{a}$, or an underline $\underline{a}$. It is represented pictorially by a **directed line segment** as shown.

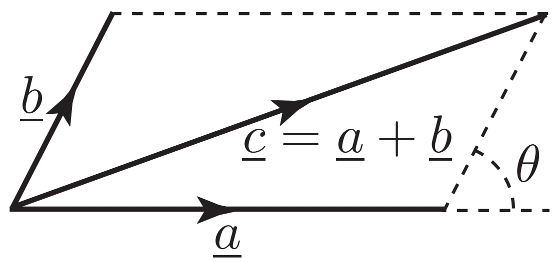
The length of the line segment represents the vector's magnitude. Its orientation, together with the arrow shown, gives the direction of the vector.



The magnitude of a vector $\underline{a}$ is written $|\underline{a}|$ or simply a. A **unit vector** has magnitude 1. $-\underline{a}$ has the magnitude of $\underline{a}$ but is opposite in direction.

Addition: The parallelogram rule defines addition of two vectors. $\underline{c} = \underline{a} + \underline{b} = \underline{b} + \underline{a}$.

$$c^2 = a^2 + b^2 + 2\,a\,b\,\cos\theta$$

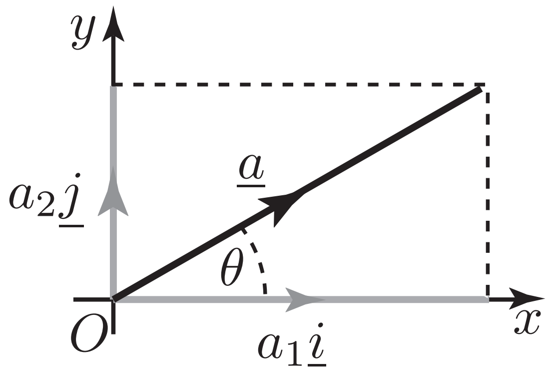


where θ is the angle between $\underline{a}$ and $\underline{b}$, as shown. $\underline{c}$ is called the **resultant** of $\underline{a}$ and $\underline{b}$.

Rectangular Components: Let $\underline{i}$ be a unit vector in the direction of the positive x axis and $\underline{j}$ be a unit vector in the direction of the positive y axis. In two dimensions the vector $\underline{a}$ can be written as the sum of two rectangular vector components:

$$\underline{a} = a_1\underline{i} + a_2\underline{j} \quad \text{or} \quad \underline{a} = (a_1, a_2).$$

The scalar components a_1 and a_2 are given by $a_1 = a\cos\theta$, $a_2 = a\sin\theta$, where θ is the angle $\underline{a}$ makes with the positive x axis. Any vector can be replaced by its rectangular vector components, starting at the same point.



Using Pythagoras' theorem it follows that $|\underline{a}| = \sqrt{a_1^2 + a_2^2}$. In a natural extension to three dimensions we can write

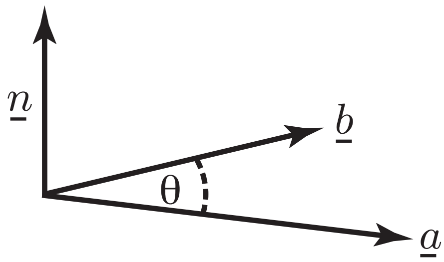
$$\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k} \quad \text{and} \quad |\underline{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$\underline{k}$ is a unit vector in the direction of the positive z axis.

Scalar (dot) product & Vector (cross) product:

If $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ and $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ then

$$\underline{a}\cdot\underline{b} = |\underline{a}|\,|\underline{b}|\cos\theta$$
$$\underline{a}\cdot\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$$
$$\underline{a}\times\underline{b} = |\underline{a}|\,|\underline{b}|\sin\theta\,\underline{n}$$



Here θ is the angle between $\underline{a}$ and $\underline{b}$, and $\underline{n}$ is a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$ in a sense defined by the right-hand screw rule.

$$\underline{a}\times\underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
$$= (a_2b_3 - a_3b_2)\underline{i} - (a_1b_3 - a_3b_1)\underline{j} + (a_1b_2 - a_2b_1)\underline{k}$$