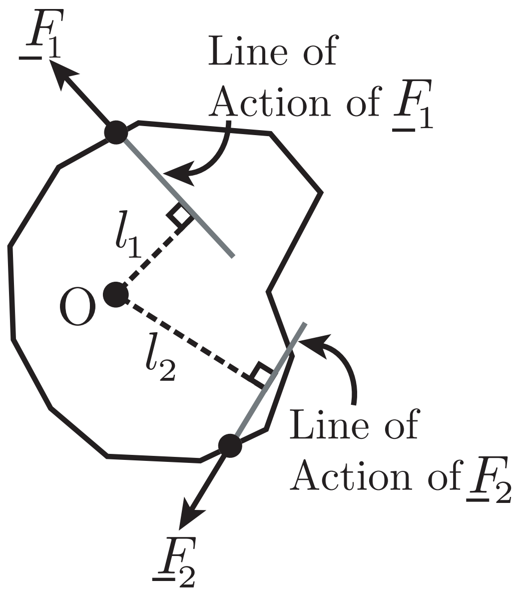
13. Rigid bodies

Consider an axis perpendicular to the plane of the paper and passing through O. The rigid body is acted upon by the forces $\underline{F}_1$ and $\underline{F}_2$, lying in the plane. $\underline{F}_1$, $\underline{F}_2$ produce anti-clockwise/clockwise rotation about the axis, respectively. By convention, anti-clockwise rotation is taken as positive. The **moments** of $\underline{F}_1$ and $\underline{F}_2$ about the axis through O are defined by



$$\Gamma_1 = +F_1\,l_1 \qquad\qquad \Gamma_2 = -F_2\,l_2$$

where l_1 and l_2 are the perpendicular distances of the lines of action of $\underline{F}_1$ and $\underline{F}_2$ from O. The line of action of a force is a line with the same orientation as the force and which passes through its point of action.

For rigid bodies there are two necessary conditions for equilibrium:

First condition: When a body is in equilibrium the resultant force, $\underline{R} = (R_x, R_y, R_z)$, of all the forces acting on it, is zero. (This condition also applies to particles.) Thus

$$\underline{R} = \underline{0} \qquad R_x = 0 \qquad R_y = 0 \qquad R_z = 0$$

where R_x, R_y and R_z are the net sums of the x, y and z scalar components of the forces, respectively.

Second condition: When a body is in equilibrium the sum of the moments, about any arbitrary axis, is zero:

$$\Sigma\,\Gamma = 0$$

Centre of mass: This is the point in a body such that an external force produces an acceleration just as though the whole mass were concentrated there. Let $(\overline{x}, \overline{y}, \overline{z})$ be the coordinates of the centre of mass of a system of particles, each of mass m_1, m_2, ..., and centres of mass located at (x_1, y_1, z_1), $(x_2, y_2, z_2), \ldots$. Then

$$\overline{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} \qquad \overline{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} \qquad \overline{z} = \frac{\Sigma m_i z_i}{\Sigma m_i}$$

from which

$$\Sigma m_i(x_i - \overline{x}) = \Sigma m_i(y_i - \overline{y}) = \Sigma m_i(z_i - \overline{z}) = 0$$

Then the sum of moments about an axis through the centre of mass is zero. Symmetry can be useful in finding the centre of mass. The centre of mass of a homogeneous sphere, circular disk or rectangular plate is at its centre.

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