Percentages

In this unit we shall look at the meaning of percentages and carry out calculations involving percentages. We will also look at the use of the percentage button on calculators.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- calculate a percentage of a given quantity;

- increase or decrease a quantity by a given percentage;

- find the original value of a quantity when it has been increased or decreased by a given percentage;

- express one quantity as a percentage of another.

Contents

1. Introduction

The word 'percentage' is very familiar to us as it is used regularly in the media to describe anything from changes in the interest rate, to the number of people taking holidays abroad, to the success rate of the latest medical procedures or exam results. Percentages are a useful way of making comparisons, apart from being used to calculate the many taxes that we pay such as VAT, income tax, domestic fuel tax and insurance tax, to name but a few.

So percentages are very much part of our lives. But what does percentage actually mean?

Now 'per cent' means 'out of 100'; and 'out of', in mathematical language, means 'divide by'. So if you score 85% (using the symbol '%' for percentage) on a test then, if there were a possible 100 marks altogether, you would have achieved 85 marks. So

$$85\% = \frac{85}{100}.$$

Let us look at some other common percentage amounts, and their fraction and decimal equivalents.

$$
\begin{array}{rcccccc}
75\% & = & \frac{75}{100} & = & \frac{3}{4} & = & 0.75 \\
50\% & = & \frac{50}{100} & = & \frac{1}{2} & = & 0.5 \\
25\% & = & \frac{25}{100} & = & \frac{1}{4} & = & 0.25 \\
10\% & = & \frac{10}{100} & = & \frac{1}{10} & = & 0.1 \\
5\% & = & \frac{5}{100} & = & \frac{1}{20} & = & 0.05.
\end{array}
$$

It is worth noting that 50% can be found be dividing by 2, and that 10% is easily found by dividing by 10.

Now let us look at writing fractions as percentages. For example, say you get 18 marks out of 20 in a test. What percentage is this?

First, write the information as a fraction. You gained 18 out of 20 marks, so the fraction is $\frac{18}{20}$. Since a percentage requires a denominator of 100, we can turn $\frac{18}{20}$ into a fraction out of 100 by multiplying both numerator and denominator by 5:

$$\frac{18}{20} \quad = \quad \frac{18 \times 5}{20 \times 5} \quad = \quad \frac{90}{100} \quad = \quad 90\%.$$

Since we are multiplying both the numerator and the denominator by 5, we are not changing the value of the fraction, merely finding an equivalent fraction.

In that example it was easy to see that, in order to make the denominator 100, we needed to multiply 20 by 5. But if it is not easy to see this, such as with a score of, say, 53 out of 68, then we simply write the amount as a fraction and then multiply by $\frac{100}{100}$:

$$\frac{53}{68} \times \frac{100}{100} \quad = \quad 53 \div 68 \times 100\% \quad = \quad 77.94\%$$

which is 78% to the nearest whole number. Although it is easier to use a calculator for this type of calculation, it is advisable not to use the % button at this stage. We shall look at using the percentage button on a calculator at the end of this unit.

 Key Point

Percentage means 'out of 100', which means 'divide by 100'.

To change a fraction to a percentage, divide the numerator by the denominator and multiply by 100%.

Exercises 1

(a) 7 out of every 10 people questioned who expressed a preference liked a certain brand of cereal. What is this as a percentage?

(b) In a test you gained 24 marks out of 40. What percentage is this?

(c) 30 out of 37 gambling sites on the Internet failed to recognise the debit card of a child. What is this as a percentage?

2. Finding percentage amounts

For many calculations, we need to find a certain percentage of a quantity. For example, it is common in some countries to leave a tip of 10% of the cost of your meal for the waiter. Say a meal costs £25.40:

$$10\% \text{ of } £25.40 \quad = \quad \frac{10}{100} \times £25.40 \quad = \quad £2.54 \,.$$

As mentioned before, an easy way to find 10% is simply to divide by 10. However the written method shown above is useful for more complicated calculations, such as the commission a salesman earns if he receives 2% of the value of orders he secures. In one month he secures £250,000 worth of orders. How much commission does he receive?

$$2\% \text{ of } £250,000 \quad = \quad \frac{2}{100} \times £250,000 \quad = \quad £5,000 \,.$$

Many things that we buy have VAT added to the price, and to calculate the purchase price we have to pay we need to find $17\frac{1}{2}\%$ and add it on to the price. This can be done in two ways.

For example, the cost of a computer is £634 plus VAT. Find the total cost.

$$
\begin{aligned}
\text{VAT} \quad &= \quad 17\tfrac{1}{2}\% \text{ of } £634 \\
&= \quad \frac{17.5}{100} \times £634 \\
&= \quad £110.95 \\
\text{so total cost} \quad &= \quad £634 + £110.95 \\
&= \quad £744.95 \,.
\end{aligned}
$$

Or, instead of thinking of the total cost as 100% of the price plus $17\frac{1}{2}\%$ of the price, we can think of it as $117\frac{1}{2}\%$ of the price, so that

$$117\tfrac{1}{2}\% \text{ of } \pounds 634 \quad = \quad \frac{117.5}{100} \times \pounds 634 \quad = \quad \pounds 744.95 \,.$$

Although $17\frac{1}{2}\%$ seems an awkward percentage to calculate, there is an easy method you can use so that you do not need a calculator. Let us look at the same example again.

$$\pounds 634$$

10%	is	£63.40	(divide by 10)
5%	is	£31.70	(half of 10%)
$2\frac{1}{2}\%$	is	£15.85	(half of 5%)

so $17\frac{1}{2}\%$ is £110.95 (add the above).

In a similar way to a percentage increase, there is a percentage decrease. For example, shops often offer discounts on certain goods. A pair of trainers normally costs £75, but they are offered for 10% off in the sale. Find the amount you will pay.

Now 10% of £75 is £7.50, so the sale price is $\pounds 75 - \pounds 7.50 = \pounds 67.50$.

What you are paying is the 100% of the cost, minus 10% of the cost, so in effect you are paying 90% of the cost. So we could calculate this directly by finding 90% of the cost.

$$90\% \text{ of } \pounds 75 \quad = \quad \frac{90}{100} \times \pounds 75 \quad = \quad \pounds 67.50 \,.$$

3. Finding the original amount before a percentage change

Let us look at an example where the price includes VAT, and we need the price excluding VAT.

Example

The cost of a computer is £699 including VAT. Calculate the cost before VAT.

Solution

Now a common mistake here is to take $17\frac{1}{2}\%$ of the cost including VAT, and then subtract. But this is wrong, because the VAT is not $17\frac{1}{2}\%$ of the cost *including* the VAT, which is what we have been given. Instead, the VAT is $17\frac{1}{2}\%$ of the cost *before* the VAT, and this is what we are trying to find. So we have to use a different method.

Now we have been told that £699 represents the cost including VAT, so that must equal the cost before VAT, plus the VAT itself, which is $17\frac{1}{2}\%$ of the cost before VAT. So the total must be $100\% + 17\frac{1}{2}\% = 117\frac{1}{2}\%$ of the cost before VAT. Thus, to find 1% we divide by $117\frac{1}{2}$. So

$$117\tfrac{1}{2}\% \text{ of the price excluding VAT} \quad = \quad \pounds 699,$$
$$1\% \text{ of the price excluding VAT} \quad = \quad \frac{\pounds 699}{117.5} \,.$$

To find the cost before VAT we want 100%, so now we need to multiply by 100. Then

$$\text{the price excluding VAT} \quad = \quad \frac{\pounds 699}{117.5} \times 100$$
$$= \quad \pounds 594.89 \,.$$

Let us look at another situation where we need to find an original amount before a percentage increase has taken place.

Example

An insurance company charges a customer £320 for his car insurance. The price includes government insurance premium tax at 5%. What is the cost before tax was added?

Solution

Here, the £320 represents 105% of the cost, so to calculate the original cost, 100%, we need to calculate

$$\frac{£320}{105} \times 100 = £304.76 \,.$$

Here is one more similar calculation, but this time there has been a reduction in cost.

Example

A shop has reduced the cost of a coat by 15% in a sale, so that the sale price is £127.50. What was the original cost of the coat?

Solution

In this case, £127.50 represents 85% (that is, $100\% - 15\%$) of the original price. So if we write this as a fraction, we divide by 85 to find 1% and then multiply by 100 to find the original price.

$$\frac{£127.50}{85} \times 100 = £150 \,.$$

Key Point

If you are given a percentage change and the final amount, write the final amount as 100% plus (or minus) the percentage change, multiplied by the original amount.

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4. Expressing a change as a percentage

We might wish to calculate the percentage by which something has increased or decreased. To do this we use the rule

$$\frac{\text{actual increase or decrease}}{\text{original cost}} \times 100\% \,.$$

So you write the amount of change as a fraction of the original amount, and then turn it into a percentage.

Example

Four years ago, a couple paid £180,000 for their house. It is now valued at £350,000. Calculate the percentage increase in the value of the house.

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Solution

$$\begin{aligned}
\text{Percentage increase} &= \frac{\text{actual increase}}{\text{original cost}} \times 100\% \\[2mm]
&= \frac{\pounds 350,000 - \pounds 180,000}{\pounds 180,000} \times 100\% \\[2mm]
&= \frac{\pounds 170,000}{\pounds 180,000} \times 100\% \\[2mm]
&= 94\% \text{ to the nearest } 1\% \, .
\end{aligned}$$

Let us look at an example where the change has been a decrease.

Example

A car cost $\pounds 12{,}000$. After 3 years it is worth $\pounds 8{,}000$. What is the percentage decrease?

Solution

$$\begin{aligned}
\text{Percentage decrease} &= \frac{\text{actual decrease}}{\text{original cost}} \times 100\% \\[2mm]
&= \frac{\pounds 12,000 - \pounds 8,000}{\pounds 12,000} \times 100\% \\[2mm]
&= \frac{\pounds 4,000}{\pounds 12,000} \times 100\% \\[2mm]
&= 33\% \text{ to the nearest } 1\% \, .
\end{aligned}$$

Key Point

To write an increase or decrease as a percentage, use the formula

$$\frac{\text{actual increase or decrease}}{\text{original cost}} \times 100\% \, .$$
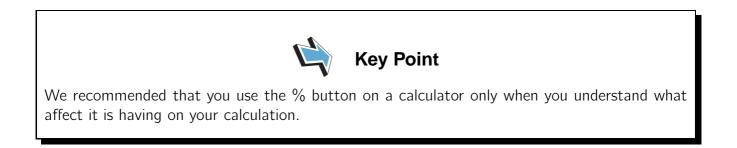
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5. Calculating percentages using a calculator

Here is a warning about using the percentage button on a calculator: the result depends on when you press the % button in your calculation. Sometimes it has no effect, sometimes it seems to divide by 100, and at other times it multiplies by 100. Here are some examples

- Pressing $48 \div 400\%$ gives an answer of 12. Now $48 \div 400 = 0.12$, so pressing the % button has had the effect of multiplying by 100. This has found 48 as a percentage of 400.

- Pressing $1 \div 2 \times 300\%$ gives the answer 1.5. Now $1 \div 2 \times 300 = 150$, so pressing the % button here has divided by 100. This has found 300% of a half.

- Pressing $400 \times 50\%$ gives an answer of 200. Now $400 \times 50 = 20,000$, so pressing $\%$ here has divided by 100. This has found 50% of 400.

- Pressing $50\% \times 400$ results in 400 on the display, requiring $=$ to be pressed to display an answer of 20,000. So pressing the $\%$ button here has had no effect.

Key Point

We recommended that you use the $\%$ button on a calculator only when you understand what affect it is having on your calculation.

Exercises 2

(a) What is the amount of VAT (at a rate of $17\frac{1}{2}\%$) which must be paid on an imported computer game costing £16.00?

(b) A visitor to this country buys a souvenir costing £27.50 including VAT at $17\frac{1}{2}\%$. How much VAT can be reclaimed?

(c) At the end of 1999 you bought shares in a company for £100. During 2000 the shares increased in value by 10%. During 2001 the shares decreased in value by 10%. How much were the shares worth at the end of 2001?

(Give your answers to the nearest penny.)

Answers

1.

(a) 70%   (b) 60%   (c) 81%

2.

(a) £2.80   (b) £4.10   (c) £99.00 .