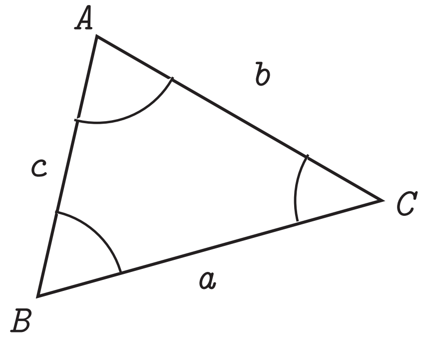
Complex Numbers



Cartesian form:

$z = a + bj$ where $j = \sqrt{-1}$

Polar form:

$z = r(\cos\theta + j\sin\theta) = r\angle\theta$

$a = r\cos\theta$, $b = r\sin\theta$, $\tan\theta = \frac{b}{a}$

Exponential form:

$z = r\mathrm{e}^{j\theta}$

Euler's relations:

$\mathrm{e}^{j\theta} = \cos\theta + j\sin\theta$, $\quad \mathrm{e}^{-j\theta} = \cos\theta - j\sin\theta$

Multiplication and division in polar form:

$$z_1 z_2 = r_1 r_2 \angle(\theta_1 + \theta_2), \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

If $z = r\angle\theta$, then $z^n = r^n\angle(n\theta)$

De Moivre's theorem:

$$(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$$

Relationship between hyperbolic and trig functions:

$\cos jx = \cosh x$, $\quad \sin jx = j\sinh x$

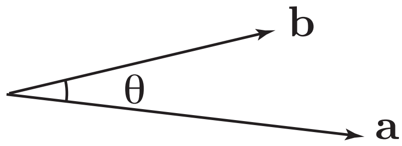
$\cosh jx = \cos x$, $\quad \sinh jx = j\sin x$

i rather than j may be used to denote $\sqrt{-1}$.

Vectors

If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ then $|\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$

Scalar product

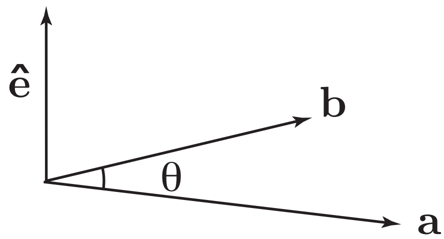


$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Vector product



$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \sin\theta\, \hat{\mathbf{e}}$

$\hat{\mathbf{e}}$ is a unit vector perpendicular to the plane containing $\mathbf{a}$ and $\mathbf{b}$ in a sense defined by the right hand screw rule.

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= (a_2 b_3 - a_3 b_2)\mathbf{i} + (a_3 b_1 - a_1 b_3)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \end{aligned}$$