Matrices and Determinants

The 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has determinant

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The 3×3 matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ has determinant

$$|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(expanded along the first row).

The inverse of a 2×2 matrix

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
provided that $ad - bc \neq 0$.

Matrix multiplication: for 2×2 matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha & \gamma \\ \beta & \delta \end{pmatrix} = \begin{pmatrix} a\alpha + b\beta & a\gamma + b\delta \\ c\alpha + d\beta & c\gamma + d\delta \end{pmatrix}$$

Remember that $AB \neq BA$ except in special cases.

The Binomial Coefficients

The coefficient of x^k in the binomial expansion of $(1+x)^n$ when n is a positive integer is denoted by $\binom{n}{k}$ or ${}^nC_k$.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \binom{n}{n-k}$$

0!=1, $\quad n! = n(n-1)!$

so, for example, $4! = 1.2.3.4$

The pattern of the coefficients is seen in

Pascal's triangle:

$$\begin{array}{ccccccccccccc}
 & & & & 1 & & 1 & & & & & & \\
 & & & 1 & & 2 & & 1 & & & & & \\
 & & 1 & & 3 & & 3 & & 1 & & & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
\vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & &
\end{array}$$

${}^nC_k$ is the number of subsets with k elements that can be chosen from a set with n elements.