Sequences and Series

Arithmetic progression:

$a, a+d, a+2d, \ldots$

$a =$ first term, $d =$ common difference,

kth term $= a + (k-1)d$

Sum of n terms, $S_n = \frac{n}{2}(2a + (n-1)d)$

Sum of the first n integers,

$1 + 2 + 3 + \ldots + n =$

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$

Sum of the squares of the first n integers,

$1^2 + 2^2 + 3^2 + \ldots + n^2 =$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$$

Geometric progression: $a, ar, ar^2, \ldots$

$a =$ first term, $r =$ common ratio,

kth term $= ar^{k-1}$

Sum of n terms, $S_n = \frac{a(1-r^n)}{1-r}$, provided $r \neq 1$

Sum of an infinite geometric series:

$S_\infty = \frac{a}{1-r}, \quad -1 < r < 1$

The binomial theorem

If n is a positive integer

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots+x^n$$

When n is negative or fractional, the series is infinite and converges when $-1 < x < 1$

Standard power series expansions

$$\mathrm{e}^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots \text{ for all } x$$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots \text{ for all } x$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots \text{ for all } x$$

$$\log_{\mathrm{e}}(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots \text{ for } -1 < x \leqslant 1 \text{ only}$$

The exponential function as the limit of a sequence

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$